

**SIR VEN**  
(*GURU ADIWIRA KEBANGSAAN 2019*)

**SPM 2023**

# **SOALAN RAMALAN** *MATEMATIK TAMBAHAN* *KERTAS 2*

**LOG/INDICES/SURD**



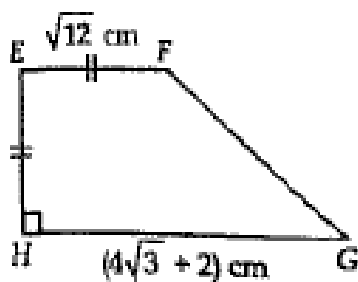


- (a) Tunjukkan bahawa  $7^{n+2} - 2(7^{n+1}) + 7^n$  boleh dibahagi tepat dengan 9 bagi semua integer positif  $n$ . [2 markah]

Show that  $7^{n+2} - 2(7^{n+1}) + 7^n$  is divisible exactly by 9 for all positive integers  $n$ . [2 marks]

- (b) Rajah 7 menunjukkan sebuah trapezium  $EFGH$ .

Diagram 7 shows the trapezium  $EFGH$ .



Rajah 7 / Diagram 7

Tentukan luas trapezium dalam bentuk surd.

Determine the area of the trapezium in the form of surd.

[3 markah]

[3 marks]

- (c) Diberi  $\log(a - 3b) = \frac{1}{2}(\log a + \log b)$ , buktikan  $a^2 + 9b^2 = 7ab$ .

Given  $\log(a - 3b) = \frac{1}{2}(\log a + \log b)$ , prove that  $a^2 + 9b^2 = 7ab$ .

[3 markah]

[3 marks]



- (a) Nisbahkan penyebut dan permudahkan.  
*Rationalize the denominator and simplify.*

$$\frac{3\sqrt{5}}{\sqrt{5} + \sqrt{3}}$$

[2 markah]  
[2 marks]

- (b) Selesaikan persamaan  
*Solve the equation*

$$4^{\log_2 x} = 64$$

[2 markah]  
[2 marks]

- (c) Wang simpanan Shahida selepas  $n$  tahun dalam sebuah bank ialah RM  $1500(1+0.08)^n$ .  
Hitung bilangan tahun minimum supaya wang simpanannya melebihi RM 20 000.  
*Shahida's savings after  $n$  years in a bank is RM  $1500(1+0.08)^n$ . Calculate the minimum number of years so that her savings exceed RM 20 000.*

[3 markah]  
[3 marks]



(a) Buktikan

*Prove*

$$\log_a mn = \log_a m + \log_a n$$

[2 markah]

[2 marks]

(b) Diberi  $\log_x 3 = p$  dan  $\log_x 5 = q$ , ungkapkan  $\log_5 81x^2$  dalam sebutan  $p$  dan  $q$ .

*Given that  $\log_x 3 = p$  and  $\log_x 5 = q$ , express  $\log_5 81x^2$  in terms of  $p$  and  $q$ .*

[3 markah]

[3 marks]



(a) Bentukkan persamaan kuadratik yang mempunyai punca-punca  $5 + \sqrt{3}$

dan  $5 - \sqrt{3}$ .

[2 markah]

*Form the quadratic equation which has the roots  $5 + \sqrt{3}$  and  $5 - \sqrt{3}$ .*

[2 marks]

(b) Satu punca persamaan kuadratik  $2x^2 - 7 = 16x - 3m$  adalah tiga kali ganda punca yang satu lagi, cari nilai  $m$ . [3 markah]

*One of the roots of the quadratic equation  $2x^2 - 7 = 16x - 3m$  is three times the value of the other root, find the value of  $m$ .* [3 marks]

(a) Diberi  $2^x = 4^y = 8^z$ , ungkapkan  $y$  dalam sebutan  $x$  dan  $z$ .

Given  $2^x = 4^y = 8^z$ , express  $y$  in terms of  $x$  and  $z$ .

[4 markah]

[4 marks]

(b) Suhu sejenis bongkah aluminium menyusut daripada  $120^\circ\text{C}$  kepada  $T^\circ\text{C}$  mengikut persamaan  $T = 100(0.95)^x$  selepas  $x$  saat. Hitungkan

The temperature of a cube of aluminium decreases from  $120^\circ\text{C}$  to  $T^\circ\text{C}$  following the equation

$T = 100(0.95)^x$  after  $x$  seconds. Calculate

(i) suhu aluminium selepas 4 saat,  
the temperature of the aluminium cube after 4 seconds,

(ii) masa,  $t$  dalam saat untuk suhu aluminium menyusut daripada  $120^\circ\text{C}$  kepada  $90^\circ\text{C}$ .  
the time,  $t$  in seconds for the aluminium to decrease from  $120^\circ\text{C}$  to  $90^\circ\text{C}$ .

[4 markah]

[4 marks]

LOG/INDICES/SURD

(a) Diberi  $\log_a N = x \Leftrightarrow N = a^x$ , nyatakan syarat bagi  $N$ .

[1 markah]

Given  $\log_a N = x \Leftrightarrow N = a^x$ , state the condition of  $N$ .

[1 mark]

(b) Diberi  $\frac{5^p}{4} = \left(\frac{5}{7}\right)^p$ , cari nilai  $p$ .

[2 markah]

Given  $\frac{5^p}{4} = \left(\frac{5}{7}\right)^p$ , find the value of  $p$ .

[2 marks]

(c) Diberi  $\log_{128} x = h$  dan  $\log_4 y = k$ , ungkapkan  $\frac{x}{y}$  dalam sebutan  $h$  dan  $k$ .

[2 markah]

Given  $\log_{128} x = h$  and  $\log_4 y = k$ , express  $\frac{x}{y}$  in terms of  $h$  and  $k$ .

[2 marks]

- (a) Ungkapkan  $\frac{9+\sqrt{3}}{3-\sqrt{3}}$  dalam bentuk  $a+b\sqrt{c}$ , dengan keadaan  $a$ ,  $b$  dan  $c$  ialah pemalar. [3 markah]

Express  $\frac{9+\sqrt{3}}{3-\sqrt{3}}$  in the form  $a+b\sqrt{c}$ , where  $a$ ,  $b$  and  $c$  are constants. [3 marks]

- (b) Tunjukkan bahawa  $\sqrt{a} + \sqrt{b} = \sqrt{a+2\sqrt{ab}+b}$ , dengan keadaan  $a$  dan  $b$  ialah nombor positif. Seterusnya ringkaskan  $\sqrt{3+2\sqrt{2}}$ . [3 markah]

Show that  $\sqrt{a} + \sqrt{b} = \sqrt{a+2\sqrt{ab}+b}$ , where  $a$  and  $b$  are positive numbers.

Hence simplify  $\sqrt{3+2\sqrt{2}}$ . [3 marks]

[2 markah]

(a) Selesaikan  $3^{2x+1} = \frac{1}{\sqrt{27^{x+4}}}$

Solve  $3^{2x+1} = \frac{1}{\sqrt{27^{x+4}}}$ .

[2 marks]

(b) Ungkapkan  $\frac{6-5\sqrt{2}}{4+3\sqrt{2}}$  dalam bentuk  $a+b\sqrt{2}$ , dengan keadaan  $a$  dan  $b$  ialah

pemalar.

[3 markah]

Express  $\frac{6-5\sqrt{2}}{4+3\sqrt{2}}$  in the form  $a+b\sqrt{2}$ , where  $a$  and  $b$  are constants. [3 marks]

(c) Diberi  $2\log_p pq = 3 - \log_p p + 5\log_p q$ , dengan keadaan  $p$  dan  $q$  ialah integer positif, tunjukkan bahawa  $p = 7q$ . [3 markah]

Given  $2\log_p pq = 3 - \log_p p + 5\log_p q$ , where  $p$  and  $q$  are positive integer, show that  $p = 7q$ . [3 marks]

LOG/INDICES/SURD

Diberi bahawa  $a$  memuaskan persamaan  $\log_m a = 2(\log_m b - \log_m 2)$  dengan keadaan  $b > 0$ ,  $m > 0$  dan  $m \neq b$ .

*It is given that  $a$  satisfies the equation  $\log_m a = 2(\log_m b - \log_m 2)$  where  $b > 0$ ,  $m > 0$  and  $m \neq b$ .*

- (a) Nyatakan nilai  $b$ . [1 markah]  
*State the value of  $b$ .* [1 mark]
- (b) Ungkapkan  $b$  dalam sebutan  $a$ . [3 markah]  
*Express  $b$  in terms of  $a$ .* [3 marks]

- (a) Diberi  $2^{3x} = 2p$ ,  $2^y = q$  dan  $2^{y+3x} = 5 + 8^x$ . Ungkapkan  $p$  dalam sebutan  $q$ .  
[3 markah]

*It is given  $2^{3x} = 2p$ ,  $2^y = q$  and  $2^{y+3x} = 5 + 8^x$ . Express  $p$  in terms of  $q$ .*

[3 marks]

- (b) Selesaikan persamaan  $m + 4 = \sqrt{m + 10}$ . [2 markah]

*Solve the equation  $m + 4 = \sqrt{m + 10}$ .*

[2 marks]

Selesaikan persamaan  $\log_2 y = 4 \log_2 2$ .

*Solve the equation  $\log_2 y = 4 \log_2 2$ .*

- (a) Nisbahkan penyebut dan permudahkan  
*Rationalize and simplify*

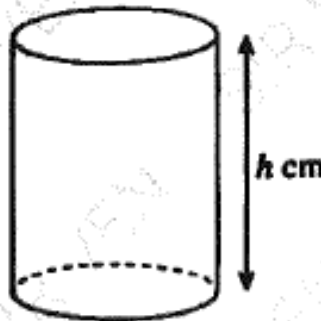
$$\frac{7\sqrt{5}}{14-\sqrt{5}}$$

[3 markah]

[3 marks]

- (b) Rajah 5 menunjukkan sebuah silinder dengan jejari  $4\sqrt{3}$  cm dan tinggi  $h$  cm. Jumlah luas permukaan silinder ialah  $56\sqrt{6}\pi$  cm<sup>2</sup>.

*Diagram 5 shows a cylinder with radius  $4\sqrt{3}$  cm and height  $h$  cm. The total surface area of the cylinder is  $56\sqrt{6}\pi$  cm<sup>2</sup>.*



Rajah 5 / Diagram 5

Cari nilai  $h$  dan beri jawapan dalam bentuk  $a\sqrt{2} + b\sqrt{3}$ , dengan keadaan  $a$  dan  $b$  adalah integer.

*Find the value of  $h$  and give your answer in the form  $a\sqrt{2} + b\sqrt{3}$ , where  $a$  and  $b$  are integers.*

[5 markah]

[5 marks]

## SABK S1

$$7^m(7^2) - 2(7^m \times 7) + 7^m$$

$$7^m(36)$$

$$\frac{1}{2} \times \sqrt{12} \times (\sqrt{12} + 4\sqrt{3} + 2)$$

$$\sqrt{3}(6\sqrt{3} + 2)$$

$$18 + 2\sqrt{3}$$

$$\log(a - 3b) = \log \sqrt{ab}$$

$$(a - 3b) = \sqrt{ab}$$

$$a^2 - 6ab + 9b^2 = ab$$

$$a^2 + 9b^2 = 7ab$$

## SABK S2

$$\frac{3\sqrt{5}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= \frac{15 - 3\sqrt{15}}{2}$$

$$4^{\log_2 x} = 64$$

$$4^{\log_2 x} = 4^3$$

$$\log_2 x = 3$$

$$x = 8$$

$$1500(1+0.08)^n > 20\,000$$

$$n > \frac{\log \frac{40}{3}}{\log 1.08}$$

$$n > 33.66$$

$$n = 34$$

## MIMS S1

$$m = a^x, \text{ maka } \log_a m = x$$

$$n = a^y, \text{ maka } \log_a n = y$$

$$mn = a^x \times a^y$$

$$mn = a^{x+y}$$

$$\log_a mn = x + y$$

$$\log_a mn = \log_a m + \log_a n$$

$$\frac{\log_x 81x^2}{\log_x 5}$$

$$\frac{\log_x 81 + \log_x x^2}{\log_x 5}$$

$$\frac{\log_x 3^4 + 2 \log_x x}{\log_x 5}$$

$$\frac{4p + 2}{q}$$

## MIMS S3

$$\text{HTP} = (5 + \sqrt{3}) + (5 - \sqrt{3}) \text{ or}$$

$$\text{HDP} = (5 + \sqrt{3})(5 - \sqrt{3})$$

$$x^2 - 10x + 22 = 0$$

$$\alpha + 3\alpha = \frac{-(-16)}{2}$$

$$\alpha = 2$$

$$\alpha(3\alpha) = \frac{-7 + 3m}{2}$$

$$m = \frac{31}{3}$$

OR

$$\alpha + \frac{1}{3}\alpha = \frac{-(-16)}{2}$$

$$\alpha = 6$$

$$\alpha\left(\frac{\alpha}{3}\right) = \frac{-7 + 3m}{2}$$

$$\frac{6^2}{3} = \frac{-7 + 3m}{2}$$

$$m = \frac{31}{3}$$

# LOG/INDICES/SURD

## JAWAPAN

### LOG/INDICES/SURD

$$2^x = 4^y$$

$$\log_{10} 2^x = \log_{10} 4^y$$

$$x \log_{10} 2 = y \log_{10} 4$$

$$\log_{10} 2 = \frac{y}{x} \log_{10} 4 \dots \dots (i)$$

$$4^y = 8^z$$

$$4^y = (2 \times 4)^z$$

$$4^y = 2^z \times 4^z$$

$$\log_{10} 4^y = \log_{10} (2^z \times 4^z)$$

$$\log_{10} 4^y = \log_{10} 2^z + \log_{10} 4^z$$

$$y \log_{10} 4 = z \log_{10} 2 + z \log_{10} 4 \dots \dots (ii)$$

Masukkan (i) ke dalam (ii)

$$y \log_{10} 4 = z \left( \frac{y}{x} \log_{10} 4 \right) + z \log_{10} 4$$

$$y = \frac{yz}{x} + z$$

$$xy = yz + xz$$

$$xy - yz = xz$$

$$y(x - z) = xz$$

$$y = \frac{xz}{x - z}$$

$$T = 100(0.95)^4$$

$$T = 81.45^\circ\text{C}$$

$$100(0.95)^4 = 90$$

$$(0.95)^4 = \frac{90}{100}$$

$$(0.95)^4 = 0.9$$

$$x \log_{10} 0.95 = \log_{10} 0.9$$

$$x = \frac{\log_{10} 0.9}{\log_{10} 0.95}$$

$$x = 2.05 \text{ saat}$$

### LOG/INDICES/SURD

(a)  $N > 0$

(b)  $p \log 5 - \log 4 = p \log 5 - p \log 7$   
 $-\log 4 = -p \log 7$

$$p = \frac{\log 4}{\log 7}$$

$$p = 0.7124$$

Atau

$$\frac{5^p}{4} = \frac{5^p}{7^p}$$

$$4 = 7^p$$

$$p = \frac{\log 4}{\log 7}$$

$$p = 0.7124$$

c)  $x = 128^h$  or  $y = 4^t$   
 $x = 2^{7h}$  or  $y = 2^{2t}$

$$\frac{x}{y} = 2^{7h-2t}$$

### LOG/INDICES/SURD

$$\frac{9 + \sqrt{3}}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$\frac{27 + 9\sqrt{3} + 3\sqrt{3} + 3}{9 - 3\sqrt{3} + 3\sqrt{3} - 3}$$

$$\frac{30 + 12\sqrt{3}}{6}$$

$$5 + 2\sqrt{3}$$

kiri:  $\sqrt{a} + \sqrt{b}$

$$(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a})^2 + 2\sqrt{a}\sqrt{b} + (\sqrt{b})^2$$

$$(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$$

$$\sqrt{a} + \sqrt{b} = \sqrt{a + 2\sqrt{ab} + b}$$

$$= \sqrt{1 + 2\sqrt{2} + 2}$$

$$= \sqrt{1} + \sqrt{2}$$

$$= 1 + \sqrt{2}$$

### LOG/INDICES/SURD

$$3^{2x+1} = 3^{-\frac{3}{2}x-6}$$

$$x = -2$$

$$\frac{6 - 5\sqrt{2}}{4 + 3\sqrt{2}} \times \frac{4 - 3\sqrt{2}}{4 - 3\sqrt{2}}$$

$$\frac{6(4) - 6(3\sqrt{2}) - 4(5\sqrt{2}) + (5\sqrt{2})(3\sqrt{2})}{4(4) - 4(3\sqrt{2}) + 4(3\sqrt{2}) - (3\sqrt{2})(3\sqrt{2})}$$

or

$$\frac{6(4) - 6(3\sqrt{2}) - 4(5\sqrt{2}) + (5\sqrt{2})(3\sqrt{2})}{4^2 - (3\sqrt{2})^2}$$

$$-27 + 19\sqrt{2}$$

$$\log_7 p^2 q^2 + \log_7 p - \log_7 q^5 = 3$$

$$\log_7 \frac{p^3}{q^3} = 3$$

$$\left(\frac{p}{q}\right)^3 = 7^3$$

$$p = 7q$$

**LOG/INDICES/SURD**

JAWAPAN



**LOG/INDICES/SURD**

(a) 1

(b)  $\log_m a = 2 \log_m \left( \frac{b}{2} \right)$

$$a = \frac{b^2}{4}$$

$$b = 2\sqrt{a}$$

**LOG/INDICES/SURD**

(a)  $2^y \times 2^{3x} = 5 + 2^{3x}$

$2pq = 5 + 2p$  atau setara

$$p = \frac{5}{2(q-1)}$$

(b)  $(m+4)^2 = m+10$  atau  $x^2 + 7x + 6 = 0$

$(x+1)(x+6) = 0$

$m = -1, -6$

**LOG/INDICES/SURD**

$\frac{1}{\log_2 y}$  atau  $\frac{1}{\log_y 2}$  P1

$(\log_2 y)^2 = 4$  atau  $(\log_y 2)^2 = \frac{1}{4}$  K1

$\log_2 y = 2$  atau  $\log_2 y = -2$  atau  $\log_y 2 = \frac{1}{2}$  atau  $\log_y 2 = -\frac{1}{2}$  K1

$4, \frac{1}{4}$  NI

**LOG/INDICES/SURD**

$$\frac{7\sqrt{5}}{14-\sqrt{5}} \times \frac{14+\sqrt{5}}{14+\sqrt{5}}$$

$$\frac{98\sqrt{5} + 7(5)}{14^2 - 5}$$

$$\frac{98\sqrt{5} + 35}{191}$$

$2\pi j h + 2\pi j^2 = 56\pi\sqrt{6}$

$2\pi(4\sqrt{3})h + 2\pi(4\sqrt{3})^2 = 56\pi\sqrt{6}$

$8\pi\sqrt{3}h + 96\pi = 56\pi\sqrt{6}$

$\sqrt{3}h + 12 = 7\sqrt{6}$

$$h = \frac{7\sqrt{6} - 12}{\sqrt{3}}$$

$$h = \left( \frac{7\sqrt{6}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \right) - \left( \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \right)$$

$$h = \frac{7\sqrt{18} - 12\sqrt{3}}{3}$$

$h = 7\sqrt{2} - 4\sqrt{3}$